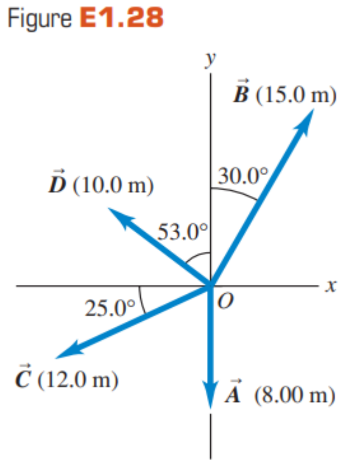
Exercise 1.49

For the vectors $\vec{A}$ and $\vec{D}$ in Fig. E1.28, (a) find the magnitude and direction of the vector product $\vec{A} \times \vec{D}$; (b) find the magnitude and direction of $\vec{D} \times \vec{A}$.

Solution



The vectors in Fig. E1.28 were written in Exercise 1.41.

$$\begin{aligned}
\mathbf{A} &= (-8.00\text{ m})\hat{\boldsymbol{j}} \\
\mathbf{B} &= (15.0 \sin 30^\circ\text{ m})\hat{\boldsymbol{i}} + (15.0 \cos 30^\circ\text{ m})\hat{\boldsymbol{j}} \\
\mathbf{C} &= (-12.0 \cos 25^\circ\text{ m})\hat{\boldsymbol{i}} + (-12.0 \sin 25^\circ\text{ m})\hat{\boldsymbol{j}} \\
\mathbf{D} &= (-10.0 \sin 53^\circ\text{ m})\hat{\boldsymbol{i}} + (10.0 \cos 53^\circ\text{ m})\hat{\boldsymbol{j}}
\end{aligned}$$

The vector (cross) product is obtained by evaluating a 3×3 determinant.

$$\begin{aligned}
\mathbf{A} \times \mathbf{D} &= \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix} \\
&= \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ 0 & -8.00\text{ m} & 0 \\ -10.0 \sin 53^\circ\text{ m} & 10.0 \cos 53^\circ\text{ m} & 0 \end{vmatrix} \\
&= \begin{vmatrix} -8.00\text{ m} & 0 \\ 10.0 \cos 53^\circ\text{ m} & 0 \end{vmatrix} \hat{\boldsymbol{i}} - \begin{vmatrix} 0 & 0 \\ -10.0 \sin 53^\circ\text{ m} & 0 \end{vmatrix} \hat{\boldsymbol{j}} + \begin{vmatrix} 0 & -8.00\text{ m} \\ -10.0 \sin 53^\circ\text{ m} & 10.0 \cos 53^\circ\text{ m} \end{vmatrix} \hat{\boldsymbol{k}} \\
&= [(-8.00\text{ m})(0) - (0)(10.0 \cos 53^\circ\text{ m})]\hat{\boldsymbol{i}} - [(0)(0) - (0)(-10.0 \sin 53^\circ\text{ m})]\hat{\boldsymbol{j}} \\
&\qquad + [(0)(10.0 \cos 53^\circ\text{ m}) - (-8.00\text{ m})(-10.0 \sin 53^\circ\text{ m})]\hat{\boldsymbol{k}} \\
&\approx 0\hat{\boldsymbol{i}} - 0\hat{\boldsymbol{j}} + (-63.9\text{ m}^2)\hat{\boldsymbol{k}} \\
&\approx -63.9\text{ m}^2\hat{\boldsymbol{k}}
\end{aligned}$$

Also,

$$\begin{aligned}
\mathbf{D} \times \mathbf{A} &= -(\mathbf{A} \times \mathbf{D}) \\
&= -(-63.9 \text{ m}^2 \hat{\boldsymbol{k}}) \\
&= 63.9 \text{ m}^2 \hat{\boldsymbol{k}}.
\end{aligned}$$

The magnitudes of $\mathbf{A} \times \mathbf{D}$ and $\mathbf{D} \times \mathbf{A}$ are both about 63.9 m^2, and they point in the negative and positive z-directions, respectively.